**Board Feet Model**

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**Problem Statement**

When cutting down trees there is a unit of measurement that helps quantify the amount of lumber that was cut, and this unit is called board feet. The purpose of this mathematical model is to be able to predict the amount of board feet that a tree has from knowing a dimension of the tree. For this problem there will be two models that are to be evaluated for solving this problem. Each model has it’s own set of assumptions and the job of this paper is to see which model is better for the estimation of board feet. These main assumptions will be discussed shortly.

**Assumptions**

The first model assumption will be referred to as *Assumption 1* and is as follows,

1. Assume all trees are right circular cylinders and the height of the tree is proportional to the diameter.

*Assumption 1* has seems to have sufficient variables in it to make a model that should be accurate enough to answer the problem statement. This assumption, intuitively makes sense due to taller trees needing a bigger base to support them due to the increase in mass. The next assumption is a more simpler assumption and is as follows,

1. Assume all trees are right circular cylinders and have approximately the same height.

*Assumption 2* on the other hand is aiming to create a model where all trees are cut at the same height. For a model with this assumption to be efficient, there must be some sort of characteristic dimension to be measured so it can be implemented in a model. These two assumptions lead to two different models to answer this problem statement. Both assumptions will now be analyzed and developed into models so that they can be compared for which one is more accurate.

**Solving Model 1**

The first model that will be addressed is *Assumption 1* where the height of the tree is proportional to the diameter of the tree. In addition to the assumptions for the geometry of the tree it is assumed that these measurements are all being taken at waist height. Now the first problem to be addressed is how the measurements of trees are going to be taken. The characteristic dimension for a tree is its diameter, but since trees are right circular cylinders this diameter is not readily available to be measured without doing additional calculations. Instead, what will be asked is for the person using the model to record the circumference of the tree in inches (Which is easily measured) and then using the following equation will give the diameter of the tree

(1)

where equation (1) will give the diameter of the tree in inches with knowing the circumference of the tree in inches beforehand. The value in equation (1) is the circumference of a tree in inches. The purpose of this model again is to figure out how much board feet is in a tree from knowing only it’s diameter. It is for this reason that equation (1) was defined so that a person using this model could measure the tree while it is still standing before it was cut. This will allow lumber cutters to make a rough estimate before actually cutting down the tree. The actual way to calculate board feet is to take the volume of an object in inches and divide it by (Source). The following equation defines this relationship

(2)

where equation (2) will give the amount of board feet from the dimensions of the object in question. The in equation (2) is the volume of the object (Which in this case it is a cylinder) and the is the board feet of the tree. So, since the object in question is a cylinder the volume for this object is

(3)

where equation (3) is the volume of the right cylinder which is the tree in this model. Substituting equation (3) into equation (2), equation (2) now becomes

(4)

where equation (4) will give the number of board feet in a tree with the dimensions of a tree. The in equation (4) is the radius of the tree and the is the height of the tree. Equation (4) does not suit well for this model because it has variables that aren’t in the diameter of the tree. First recognizing that the radius is half the diameter , equation (4) then changes to

(5)

with the equation for board feet now being in terms of the diameter the only think left to do is to create and equation that only has the diameter as a variable. It is stated in *Assumption 2* that the height of a tree is proportional to the diameter of a tree. With this the following equation can be defined

(6)

now giving the height of a tree in terms of its diameter. With this new knowledge equation (5) becomes

(7)

which will simplify to

(8)

with equation (8) now giving the amount of board feet in a tree in terms of it’s diameter. The following definition is made to make equation (8) more user friendly

(9)

where equation (8) now becomes

(10)

now giving a more aesthetically pleasing equation with equation (10). Both σ and *k* are constants in equation (10) where *k* is the proportionality constant for the model. Equation (10) can also show that the number of board feet in a tree is proportional to the cube of its diameter. This model is now ready to be verified and tested.

**Verifying Model 1**

Now that model 1 has its equation that is dependent upon its diameter, the testing of this model is now required to see how accurate it is. The first thing that must be done is trying to figure out the constant of proportionality for model 1. This can be done by graphing the number of board feet versus the diameter of the tree in inches cubed. Essentially, equation (10) must be put into excel so that the constant of proportionality can be determined. The following table has the data that was used to create the graph of equation (10)

Table 1

|  |  |  |  |
| --- | --- | --- | --- |
| **Diameter (in)** | **Actual**  **(Bd. Ft)** | **d^3\*σ** | **σ** |
| 17 | 190 | 26.8 | 0.0055 |
| 19 | 250 | 37.4 |  |
| 20 | 320 | 43.6 |  |
| 23 | 570 | 66.4 |  |
| 25 | 710 | 85.2 |  |
| 28 | 1130 | 119.7 |  |
| 32 | 1230 | 178.7 |  |
| 38 | 2520 | 299.3 |  |
| 39 | 2590 | 323.5 |  |
| 41 | 2940 | 375.9 |  |

with the data from Table 1, the following plot was created using excel

Figure 1

where Figure 1 shows the data of the actual board feet against being graphed. The resulting slope (Which can be seen in the top left of Figure 1) is the value found in equation (10). For simplicity reasons the value was rounded off to 8.0 since it was close enough to it already. Equation (10) now becomes

(11)

giving an equation that can be graphed to show how accurate this model is. Equation (11) was then graphed with the data for the diameter in Table 1 and the following plot was created

Figure 2

where Figure 2 shows the model for board feet and the actual board feet from the raw data. The purpose of Figure 2 is to show qualitatively how well this model gets the board feet from diameter. The following table shows the data that came from this model

Table 2

|  |  |  |  |
| --- | --- | --- | --- |
| **Diameter (in)** | **Model**  **(Bd. Ft)** | **Actual**  **(Bd. Ft)** | **PRE**  **%** |
| 17 | 214.3 | 190 | -12.8% |
| 19 | 299.2 | 250 | -19.7% |
| 20 | 349.0 | 320 | -9.1% |
| 23 | 530.7 | 570 | 6.9% |
| 25 | 681.6 | 710 | 4.0% |
| 28 | 957.5 | 1130 | 15.3% |
| 32 | 1429.3 | 1230 | -16.2% |
| 38 | 2393.5 | 2520 | 5.0% |
| 39 | 2587.5 | 2590 | 0.1% |
| 41 | 3006.3 | 2940 | -2.3% |

Where the data from Table 2 is showing the modeled values and the actual values. The Percent Relative Error (PRE) as seen in the far-right column of Table 2 fluctuates throughout the data set from negative to positive and then back again. The error that is seen in this model is at times very high (*d* = 17,19,32) and sometimes very low (*d* = 25,39,41). There doesn’t seem to be much of a pattern for this model but at times it is very accurate. When the PRE is negative, this indicates that the model is predicting more board feet to be in the tree when in reality there is less. This discrepancy more than likely comes from the fact that trees aren’t perfect cylinders and can’t be harvested for lumber as efficiently as a perfect cylinder. How this model should be used will be discussed in the implementing section.

**Implementing Model 1**

The use of this model is practical once the diameter of the tree is known. The errors that were seen in Table 2 show that at times the model is predicting more than and sometimes less than the actual amount of board feet in a tree. And at times the model predicts very closely to what the actual number of board feet in the tree. For this model to be used, the following equation should be given to the lumber company that cuts trees,

(12)

is the diameter of the tree and must be in inches,

is a constant and has a value of,

And once this knowledge is stated to the lumber company wanting to use this model they should be notified of why there are discrepancies in the model from what they are going to get. The following warning should be issued to the lumber company wanting to implement this model.

“This mathematical model was built off the assumption that all the trees being cut were perfect right cylinders. Trees also have significant loss from the fact that they aren’t right cylinders so not all of its volume is being harvested for lumber. This model was also shown to predict less than what the actual number of board feet was. Be leery when using this model because it is in that, purely a model. Knowing this, when using this model keep in mind that equation (12) is an estimate and will more than likely put whoever is using this model within what the actual board feet harvest will be.”

This model is purely an estimate for the number of board feet in a tree harvest. The maintaining of this model will be done later so that *Assumption 2* can be looked and compared to *Assumption 1.*

**Solving Model 2**

*Assumption 2* refers to trees being cut at the same height. For this model to be solved the same equations from model 1 can be used up to equation (5). For this model to be used a common height must be set so that the trees are being cut consistently for this model’s assumptions to be tested. So, for the sake of testing this model the uniform height to be tested is going to be 20 feet, or to be consistent with the equations units, 240 inches. Equation (5) quickly turns into

(12)

which can be simplified first with the substitution of

(13)

where equation (12) then becomes

(14)

with equation (14) now representing model 2. The variables of *B* and *d* are the same from model 1 where they represent the amount of Board Feet in board in feet and the diameter of the tree in inches respectively. It is assumed that the diameter is still found first by getting the circumference of the tree and using equation (1) to solve for the diameter. Equation (14) is now ready to be tested for how accurate it is in predicting board feet.

**Verifying Model 2**

Equation (14) represents this new model that is now to be tested. The following data is the data that was used in excel to test this model with the assumption that all the trees were at the same height.

Table 3

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Height (in)** | **Diameter (in)** | **Diameter^2 (in^2)** | **α** | **Model (Bd. Ft)** | **Actual**  **(Bd. Ft)** | **PRE**  **(Bd. Ft)** |
| 240 | 17 | 289 | 1.31 | 378.3 | 190 | -99.1% |
| 240 | 19 | 361 |  | 472.5 | 250 | -89.0% |
| 240 | 20 | 400 |  | 523.6 | 320 | -63.6% |
| 240 | 23 | 529 |  | 692.5 | 570 | -21.5% |
| 240 | 25 | 625 |  | 818.1 | 710 | -15.2% |
| 240 | 28 | 784 |  | 1026.3 | 1130 | 9.2% |
| 240 | 32 | 1024 |  | 1340.4 | 1230 | -9.0% |
| 240 | 38 | 1444 |  | 1890.2 | 2520 | 25.0% |
| 240 | 39 | 1521 |  | 1991.0 | 2590 | 23.1% |
| 240 | 41 | 1681 |  | 2200.4 | 2940 | 25.2% |

Table 3 shows the data that was used to create the following graphs of the model and the actual data. From the PRE column on the far right of Table 3 it is evident that this model does not do a good job estimating the number of board feet. There is no constant of proportionality that can be seen in equation (14) since there are no proportionality assumptions in *Assumption 2.* Because of this there will be no need for a transformed data graph. The following graph has the model’s prediction and the actual board feet for *Assumption 2.*

Figure 3

The model for *Assumption 2* has a lot higher PRE values in the entire range of data than that of model 1. The only time that that model 2 seems to be accurate is in the diameter range of 28 to 32 inches. Now that the model has been tested with data, the next step is to discuss how this model should be used.

**Implementing Model 2**

Model 2 came out with a lot higher percent relative error compared to that of model 1. When this model is to be used, it is assumed that all the trees that were going to be cut were 20 feet tall. As the diameter of the tree grew the accuracy of the model grew as well. But even at these bigger diameters the accuracy of the model is still not great. In short, this model should not be used to predict the number of board feet that is going to be available in the tree once it is harvested. With the data that was given for the trees it was more than likely true that they weren’t all the same height. Thus, why they had different diameters than one another. The bottom line is that the accuracy of this model is not good enough to be used by someone wanting to predict board feet.

**Maintaining The Models**

Model 1 gives a fair prediction for the board feet that resides in the tree. Other than a couple of diameters that lead to discrepancies in the data from the actual board feet. The only maintaining that needs to be done to model 1 is that it should be noted that an error of around 5-10% should be expected from the model. All mathematical models are not 100% accurate but letting the users know what range of error they should expect will help the practical use of this model. Model 2 is too inaccurate to use for figuring out the board feet in a tree and thus can’t be maintained to be made better with the data that was given.

**Conclusion**

Through construction of models it was confirmed that *Assumption 1* was more accurate at predicting the amount of board feet that is in a tree. The reason for this is because *Assumption 1* has more variables in it than *Assumption 2.* The model that was constructed for *Assumption 1* was more accurate than *Assumption 2’s* model because it took more characteristic dimensions into account for the model. It turned out in model 1 that the board feet of a tree was proportional to the cube of it’s diameter. In model 2, it was proportional to the square of it’s diameter and that doesn’t make sense with the units that are involved with board feet. It is from these facts that model 1 is the more accurate model for answering which assumptions lead to a better model.

# Works Cited

Source, Wood Workers. *Wood Workers Source*. 1 January 2018. Web Site. 18 February 2018. <https://www.woodworkerssource.com/shop/calc.html>.